# **Investigating the Role of Rate Limiting** in Pilot-Induced Oscillations

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From the Wright Flyer to fly-by-wire, the phenomenon of pilot-induced oscillations (PIO) has been observed on prototype, experimental, and operational military and commercial aircraft. The introduction of irreversible control systems with surfaces driven by powered actuators brought many benefits along with increased system complexity and the introduction of additional nonlinearities. Chief among these nonlinearities are the hardware and software rate limits associated with the control surface actuators. Basic sizing tradeoffs conducted in the design process set the maximum rate of an actuator, whereas software rate limits are introduced to prevent overdriving the control surface when loads or structural limitations exist. It is demonstrated that, when operating as intended, there are usually no ill effects associated with rate limits; however, certain conditions can lead to a highly saturated condition. This results in the sudden introduction of significant added phase lags to the pilot-vehicle system. In many cases, the end result is often PIO or other related loss of control events. One of the earliest well-documented PIO events involving rate limiting occurred on the first flight of the X-15. This event is significant in that common linear systems analysis techniques do not reveal a susceptibility to PIO. The analysis of X-15 flight 1–1–5 and the results of a more recent flight research program are presented in detail.

#### Nomenclature

A = actuator command input amplitude

 $a_n$  = normal acceleration

e = actuator model error signal,  $\delta_c - \delta$ 

 $e_L$  = saturation point

 $F_{es}$  = cockpit stick force

h = altitude

 $K^*$  = rate limiting element ratio of triangle output amplitude to

sinusoidal maximum input amplitude,  $(x_o/x_{i \text{ max}})$ 

p = roll rateq = pitch rate

T = linear system time constant,  $1/\omega_a$ 

 $T_{
m NL}$  = nonlinear system indicial response time constant  $t_D$  = RLE phase difference between triangle output and

sinusoidal input,  $t_o - t_i$ 

t<sub>i</sub> = RLE time to maximum input amplitude t<sub>o</sub> = RLE time to maximum output amplitude

 $V_i$  = indicated airspeed  $V_L$  = actuator rate limit  $V_V$  = vertical speed

 $x_i$  = RLE sinusoidal input amplitude

 $x_{i \max}$  = RLE maximum sinusoidal input amplitude

 $x_o = RLE$  triangle output amplitude

 $\alpha$  = angle of attack  $\beta$  = angle of sideslip

 $\Delta \phi$  = additional phase lag due to rate limiting

 $\delta$  = actuator position  $\delta_a$  = aileron position

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S<sub>c</sub> = actuator position command = horizontal stabilizer position

 $\delta$  = actuator rate  $\theta$  = pitch attitude

 $\tau_{n\theta}$  = pitch attitude phase delay

 $\omega$  = frequency

 $\omega_a$  = actuator bandwidth frequency

 $\omega_{\rm BW\theta}$  = pitch attitude bandwidth frequency, defined as frequency for the lower of 6-dB gain margin

or 45-deg phase margin

 $\omega_i$  = input frequency

 $\omega_{\text{onset}}$  = saturation onset frequency

 $\omega_{\text{PIO}}$  = pilot-induced oscillation frequency  $\omega_{u\theta}$  = pitch attitude neutral stability frequency

## Introduction

## **Pilot-Induced Oscillations and Rate Limiting**

THROUGHOUT the first 100 years of powered flight, aircraft handling qualities have often been addressed as an afterthought of the design process. Because of their often catastrophic nature, however, nothing brings attention to handling qualities as does a high-profile pilot-induced oscillation (PIO). Traditionally, the occurrence of such an event has led to significant research activities that are intended to alleviate the problem once and for all. Despite significant technical advances in this area, PIOs continue to occur with both flight-test and operational aircraft.

The focus of this paper is on the role of rate limiting in PIO. The earliest well-documented PIO event that featured rate limiting was the first flight of the X-15 aircraft in 1959. This event, which is discussed in more detail later in this paper, is one of the most analyzed of all PIO events. Among the earliest is the analysis documented in Ref. 1 that features the development of describing function approximations for a rate-limited actuator and an inverse describing function technique used to predict limit-cycle oscillations and their frequencies. This work inspired the further development of the rate limiting describing function analyses of Klyde et al.<sup>2</sup> that was conducted as part of the U.S. Air Force Unified Pilot-Induced Oscillation Theory program of the mid-1990s and is partially reported on herein.

Attention to the effects of rate limiting and other system nonlinearities surfaced again in conjunction with the severe PIOs that

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occurred with the YF-12 aircraft during aerial refueling. An excellent description of the nonlinear analysis of these events is contained in Ref. 3. More recent highly documented PIO events that involved rate limiting include the YF-22 (Ref. 4) and JAS-39 (Ref. 5) events. These events, which resulted in significant damage to the YF-22 and the loss of two JAS-39 aircraft, inspired a new thrust in handling qualities research that emphasized comprehension, prevention, and alleviation of the nonlinear effects associated with rate limiting. Specifically, the YF-22 event led to the mentioned U.S. Air Force program, whereas the JAS-39 events led SAAB to develop a patented control scheme designed to alleviate the effects of rate limiting. This scheme is described briefly later in this paper.

## **Categorizing PIO**

In 1997, a summary report by a National Research Council (NRC) Committee on the Effects of Aircraft–Pilot Coupling on Flight Safety was published.<sup>6</sup> The NRC Committee separated PIOs by category, which essentially depended on the degree of nonlinearity in the event.

- 1) Category 1 include linear pilot-vehicle system oscillations. These PIOs result from identifiable phenomena such as excessive time delay, excessive phase loss due to filters, improper control/response sensitivity, etc. They are the simplest to model, understand, and prevent. They are also the least common in operational flying.
- 2) Category 2 include quasi-linear events with some nonlinear contributions, such as rate or position limiting. For the most part, these PIOs can be modeled as linear events, with an identifiable nonlinear contribution that may be treated separately. The most common nonlinear contribution is the subject of this paper: rate limiting of a control effector actuator.
- 3) Category 3 include nonlinear PIOs with transients. Such events are difficult to recognize and rarely occur, but are always severe. Mode switching, which cannot be represented by a quasi-linear equivalent, is the common culprit.

Since the publication of the NRC's findings, several researchers have suggested that there may be other categories that are distinct from the three just defined. Category 4 PIO sometimes refers to events that are caused by, or have as a major contributor, structural modes and their interactions with the pilot.

## **PIO Trigger**

For PIO to occur, there must be a trigger. As a result of the well-publicized crashes of the YF-22 and the JAS-39 aircraft in the early 1990s it was speculated that rate limiting was a trigger for most PIOs on fly-by-wire aircraft. After the research efforts of the 1990s, we can conclude that rate limiting can be a trigger for PIO. On the other hand, sometimes PIO can be the cause of the rate limiting. In addition, we also know that it is possible to encounter severe rate limiting without a PIO. All of these possible outcomes are illustrated later in this paper.

## **Nature of Rate Limiting**

## Simplified Actuator Model with Rate Limiting

A simplified model of a rate-limited actuator that was previously analyzed<sup>2,7</sup> is shown in Fig. 1. As long as the error signal e remains below the saturation point  $e_L$ , the system behaves as a linear first-order lag whose response is entirely dictated by the time constant  $T=1/\omega_a$ , where  $\omega_a$  is the actuator bandwidth. When saturation occurs, the surface will move at its maximum rate  $V_L$ , until the

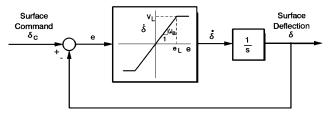


Fig. 1 Simplified actuator model.

commanded magnitude, frequency, or both are reduced. Saturation occurs when the error signal exceeds the saturation point, that is,  $e > e_L = V_L/\omega_a$ . It is also convenient to define a nonlinear system time constant,  $T_{\rm NL} = A/V_L$ , where A is the amplitude of the input command. Although this time constant applies only to the indicial (step) response describing function, it has been shown to have an important role in characterizing the nonlinear system for other input forms.<sup>2</sup>

Figure 2 shows the closed-loop actuator time responses for a saturation case with a time constant of T = 0.05 s, a saturation point of  $e_L = 2$  deg, and a sinusoidal input with an amplitude of 15 deg. The error signal still appears more or less sinusoidal, even though the 2-deg saturation point is generally exceeded throughout the run

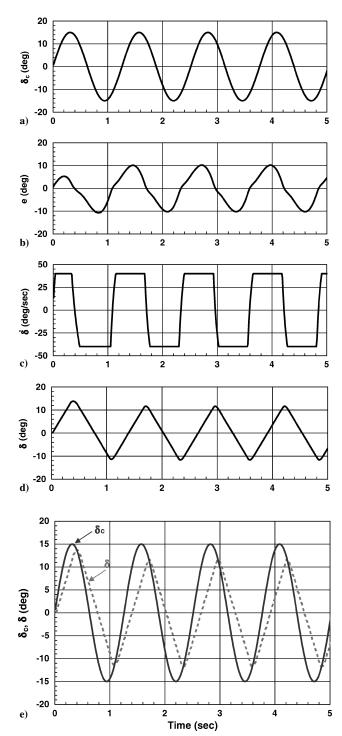
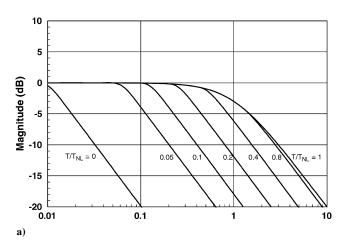


Fig. 2 Highly saturated rate-limited actuator model responses to a sinusoidal input.

length. The nonlinear nature is more evident in the actuator rate response that appears box carlike for this highly saturated case. In Fig. 2e, the actuator output position is compared to the input command. A triangle wave output response is displayed that reverses when approximately equal to the input, that is, when the error signal passes through zero. For the linear case, the delay between the output and input is the linear time constant or 0.05 s. In the nonlinear case shown in Fig. 2, this delay has increased to 0.15 s.

#### **Exact Describing Function Representation**

In general, rate saturation results in an amplitude reduction and a significant added phase lag. These characteristics are displayed in the magnitude and phase curve families of Fig. 3, which represent exact sinusoidal describing function representations of the simplified model of Fig. 1.<sup>2,7</sup> Both sets of curves are plotted as a function of normalized frequency  $\omega/\omega_a$  and the linear to nonlinear time constant ratio  $T/T_{\rm NL}$ . In the sinusoidal input case,  $T/T_{\rm NL}$ is equal to  $V_L/(\omega_a A)$ . Thus, Fig. 3a displays the describing function magnitude and Fig. 3b the phase of the nonlinear system in terms of the actuator design parameters ( $V_L$  and  $\omega_a$ ) and the input parameters (A and  $\omega$ ), all known quantities. There are several observations to note from Figs. 3. First, the  $T/T_{\rm NL}=1$  curve represents the linear case. Second, the more highly saturated cases, represented by the smaller time constant ratio curves, depart from the linear curve at a normalized frequency that is equivalent to their time constant ratio. For example, the  $T/T_{\rm NL} = 0.1$  curve departs from the linear curve at a normalized frequency of 0.1. Another more significant result is that known design and input parameters can be used to identify the added phase lag due to a rate-limiting actuator.



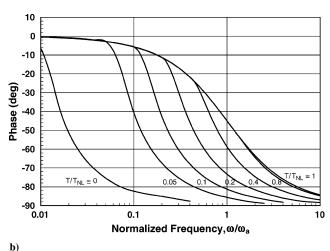


Fig. 3 Frequency response of simplified nonlinear actuator model.

#### Software Rate Limit

Often software rate limits are placed in series with an actuator to ensure that the actual physical limit is not encountered thus preventing potential damage to the flight hardware. It is not unusual for these nonlinearities to be found in the feedback path as well. As described in Ref. 2, the primary difference between the software limit and the hardware limit is the significant loss in actuator bandwidth that accompanies the hardware limit when saturated. A software rate limit can be represented by the actuator model described earlier, where the bandwidth is effectively infinite, that is,  $T/T_{\rm NL} \rightarrow 0$ . Although Fig. 3 shows both an amplitude reduction and added phase lag due to rate limiting, it is the added phase lag that is the dominant effect. Thus, when encountered in an automatic control system or a pilot–vehicle system, the primary effect of actuator or software rate saturation is to consume available phase margin that can then lead to loss of control.

A describing function approximation of a software rate limiting element can be obtained by assuming a sinusoidal-input/triangle-output rate-limiting element as was done by Hanke<sup>8</sup> and is shown in Fig. 4. This method does not consider the servoloop explicitly because only the waveforms of the input and output of the rate-limiting element are considered. It does, however, implicitly take into account the servoloop by requiring a reversal of the output whenever the servoerror,  $e = x_i - x_o$ , becomes zero. This nonlinear element model is exact when applied to a control system software limiter that contains no dynamics. It is also used to approximate an infinite bandwidth actuator, that is,  $\omega_a \gg \omega$ .

The actual output/input magnitude is defined by taking the ratio of the constant output to maximum input rates and then solving for  $x_o/x_{i \max}$ . When  $K^*$  is defined as  $x_o/x_{i \max}$ , this yields the following result:

$$K^* = x_o/x_{i \text{ max}} = (\pi/2)(\dot{x}_o/\dot{x}_{i \text{ max}})$$

This result can be written in terms of the Fig. 1 variables, by recognizing that the output rate when saturated is  $V_L$  and the maximum input rate is  $A\omega$ ,

$$K^* = (\pi/2)(V_L/A\omega)$$

The  $K^*$  parameter is next used to define the describing function magnitude and phase. The describing function magnitude is obtained by multiplying  $K^*$ , which represents the actual peak magnitude of the triangle wave, by the Fourier fundamental of the triangle wave, that is,  $8/\pi^2$ , as

$$|\delta(j\omega)/\delta_c(j\omega)| = (8/\pi^2)K^* = (4/\pi)(V_L/A\omega)$$

The phase difference between the output and input is represented by  $t_D$  in Fig. 4. It is also noted in Fig. 4 that the input and output amplitudes are equal at  $t = t_i + t_D$ . Thus, to obtain  $t_D$ , set the input

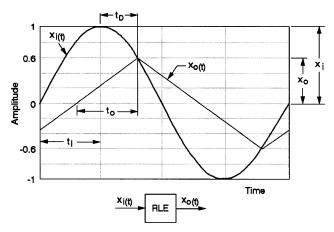


Fig. 4 Rate-limiting element time response for a sinusoidal input (from Ref. 8).

relation equal to the output and use the earlier substitution for t to produce the following:

$$x_i \sin[\omega(t_i + t_D)] = x_o$$

This equation is simplified by substituting  $K^*$  for  $x_o/x_{i \max}$ , expanding  $\sin[\omega(t_i + t_D)]$ , and noting that  $\omega t_i = \pi/2$ . This results in

$$\cos(\Delta\phi) = K^*$$

where  $\Delta \phi = \omega t_D$  is the phase angle between the input and output. Finally, the phase difference  $(\Delta \phi)$  is obtained by solving for the argument

$$\Delta \phi = \cos^{-1}(K^*)$$

The  $K^*$  parameter also provides a measure of the severity of the rate limiting, where  $0 \le K^* \le 1$ . As  $K^* \to 1$ , rate limiting diminishes and the nonlinear system becomes increasingly linear. As  $K^* \to 0$ , rate limiting increases and the corresponding amplitude reduction and added phase lag also increase as shown in Fig. 5.

#### X-15 Revisited

#### **Program Description**

The X-15 program began in 1952 as a joint venture between the military and NACA to investigate the basic problems associated with human space flight. As described in Ref. 9, the objectives of the aircraft program were to investigate aerodynamic forces, heating, stability and control, reentry characteristics, and human physiology

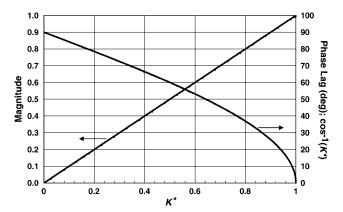


Fig. 5 Sinusoidal-input/triangle-output describing function.

at extremely high speeds and altitudes. The North American Aviation built X-15, shown in Fig. 6, was released from a B-52 carrier vehicle at 45,000 ft. The aircraft would accelerate to speeds from Mach 2 to 6 and achieve altitudes as high as 350,000 ft. The program concluded in 1968 after almost 200 flights that provided over 18 h of high-speed research.

## Flight 1-1-5 Landing Flare PIO

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The X-15 landing flare PIO occurred on 8 June 1959 with pilot Scott Crossfield at the controls. This first flight (designated flight 1–1–5) was an unpowered glide flown using the side-located controller and with the pitch damper off. Additional details of the flight and subsequent changes to the aircraft are provided in Ref. 10. As shown in the flare time history traces of Fig. 7, severe longitudinal oscillations developed near the end of the flap cycle and rate limiting is clearly evident in the horizontal stabilizer angle  $\delta_h$  trace. The triangle-wave response of the  $\delta_h$  time trace in the PIO region indicates that the actuator was operating in the highly saturated region. From the pitch rate q trace, a PIO frequency of approximately 3.3 rad/s is estimated. For this flight, the maximum control surface rate was limited to 15 deg/s.

Because the PIO occurred with the pitch stability augmentation system (SAS) off, the X-15 bare airframe data of Ref. 11 could be used to generate relevant longitudinal transfer functions. Corrections to the Ref. 11 data were made to accommodate the PIO flight condition and aircraft weight. A first-order model for the horizontal stabilizer actuator ( $\omega_a = 25 \text{ rad/s}$ ) was obtained from Ref. 12. With use of these data, the Bode and Nichols frequency-response survey of Fig. 8 for the  $\theta/\delta_h$  transfer function was generated. The transfer function gain was arbitrarily set so that the frequency response would pass through 0 dB at -110 deg of phase. Several key category 1 PIO indicators, for example, bandwidth frequency, phase delay, and average phase rate, are identified in Fig. 8. Not only do all of the applied category 1 criteria indicate that the X-15 would not be susceptible to PIO, but also the aircraft was found to be level 1 for most of the applied handling qualities measures. Figure 8 also indicates that the instability frequency for the linear system with a synchronous pilot-loop closure is 5.31 rad/s. This is almost twice that of the observed PIO frequency of 3.3 rad/s.

## **Impact of Rate Limiting**

A complete describing function analysis of the X-15 PIO is provided in Refs. 2 and 7 and is, therefore, not included here. Instead the impact of rate limiting on the airplane bandwidth handling-qualities/PIO criteria<sup>13</sup> is illustrated. Rate limiting will occur when



Fig. 6 X-15 Aircraft (NASA photograph).

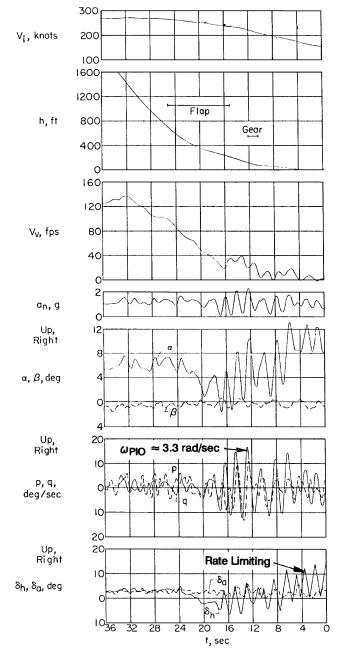


Fig. 7 Time history of the X-15 flight 1–1–5 landing flare (from Ref. 10).

the input frequency exceeds what Klyde et al.<sup>2</sup> refer to as the "saturation frequency" and what Hanke<sup>8</sup> refers to as the

$$\omega_{\rm onset} = (2K^*/\pi) \,\omega_i$$

When it was noted that  $K^*$  is a function of input amplitude A and maximum rate  $V_L$ , onset frequencies were computed for the X-15 example where  $V_L=15$  deg/s,  $\omega_i=3.3$  rad/s, that is, the PIO frequency, and A=3,6,9,12, and 15 deg. The results are plotted in Fig. 9 with a simplified version of the Ref. 13 PIO boundaries included. When A=3 deg, the onset frequency is above the PIO frequency and no rate limiting occurs. Thus, the bandwidth and phase delay parameters represent the linear system values for this case. As the input amplitude increases to 6 deg, the onset frequency moves below the PIO frequency. Here, we are in what is referred to in Ref. 2 as the near-saturation region and the effect of the rate limiting is minimal as evident by the small shift in the bandwidth/phase delay parameter plane. For the remaining three input amplitude cases, the aircraft has entered the highly saturated region and is clearly

susceptible to PIO. Note the significant jump in phase delay for these points that results from the significant added phase lag from the rate limiting. Beyond A = 9 deg, the bandwidth frequency is determined by the gain margin frequency and, thus, also shows a dramatic shift. The impact of the rate limiting is to greatly reduce bandwidth frequency and increase phase delay, both contributing factors to a poor handling aircraft that is also highly susceptible to PIO.

## **HAVE LIMITS Flight-Test Program**

#### Description

In 1997, students of the U.S. Air Force Test Pilot School at Edwards Air Force Base, California, conducted a project called HAVE LIMITS. <sup>14</sup> This student project provided the first systematic, quantitative information on the interchange between rate limiting and PIO. Three pilots flew a total of nine sorties on a variable-stability NT-33A, operated for the U.S. Air Force by Calspan (now Veridian). The experiment focused entirely on the pitch axis, with three basic aircraft models and seven levels of actuator rate limiting from 10 to 157 deg/s. The task was a pitch-and-roll attitude tracking task using commands displayed on a head-up display (HUD).

#### **HAVE LIMITS Configurations**

Three configurations from the HAVE LIMITS program illustrate the interaction of rate limiting and aircraft dynamics on the occurrence of PIO.

#### Configuration 2D

This configuration was designed with a very high pitch attitude bandwidth and low phase delay without augmentation. In the flight tests, it was PIO proof until the actuator rate was extremely low. Selected signals for a 20-s run segment from a HUD attitude tracking task are shown in Fig. 10 for configuration 2D with an actuator rate limit of 20 deg/s. The pilot for this run considered the combination of dynamics and rate limiting to be very good, with an assigned handling qualities rating (HQR) of 2 and PIO rating (PIOR) of 2. The baseline (no-limit) case was rated a 4 and a 2, respectively.

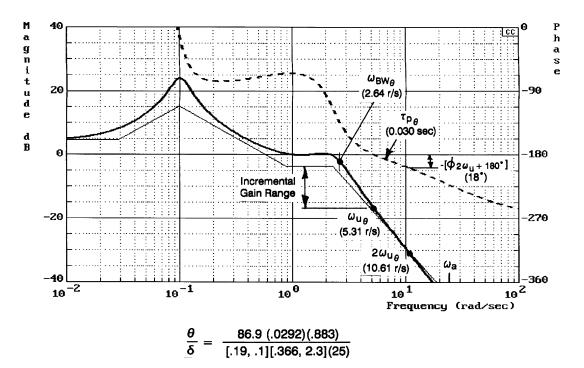
In Fig. 10a, the responses are cockpit stick force  $\vec{F}_{es}$ , in pounds and in Fig. 10b pitch rate q, in degrees per second. Figure 10c shows derived rate for the commanded and actual actuators, in degrees per second. Figure 10d shows commanded and actual elevator positions, both in degrees of surface. (The actual surface was a simulated elevator for the configuration, modeled in the variable-stability system.) Only momentary differences between commanded and actual actuator position and rate can be observed in Fig. 10. This configuration was sufficiently high bandwidth that the pilot could control it with small, short control inputs that seldom reached the limit. Rate limiting actually served to lessen the initial abruptness of the configuration.

The pitch rate and stick force traces show no real evidence of a sustained oscillation, or of out-of-phase character. There is no hint of PIO. This is significant because configuration 2D represents an airplane that is made to be level 1 by inherent aerodynamics, not with feedbacks.

## Configuration 2P

Configuration 2P, created by inserting a prefilter on configuration 2D, had deficiencies even with no limiting and received PIORs of 4 and 5 for the lowest rate limits. The time history in Fig. 11 is for a run with configuration 2P that received an HQR of 7 and a PIOR of 4. This pilot rated the no-limit case a 4 and a 2, respectively, and so rate limiting has obviously increased the tendency for PIO.

Evidence of an incipient PIO can be seen in the stick force and pitch rate traces in Fig. 11 between about 9 and 13 s. The large aft stick input at 8 s drives the actuator onto its limit, and the result is a cycle of an oscillation where stick force and pitch rate are clearly out of phase with each other. This does not develop into a full-blown PIO, however, either because the task demands were lower or because the pilot backed out of the control loop. Nevertheless, there is flight evidence of a potential for PIO.



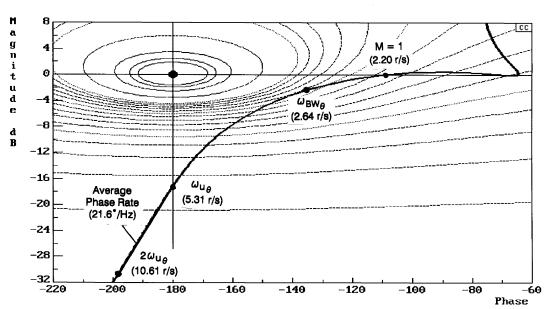


Fig. 8 X-15 pitch attitude frequency response at the flight 1–1–5 landing flare condition.

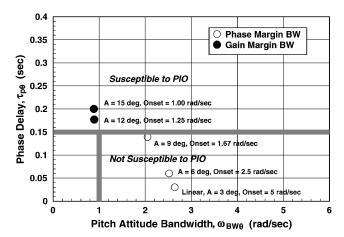


Fig. 9 X-15 example bandwidth/phase delay variations with increasing rate saturation.

Configuration 2DU

With augmentation, configuration 2DU was identical to the PIOresistant configuration 2D. However, the good dynamics were obtained by wrapping the augmentation around a bare airframe that was very unstable with an aperiodic divergence in the short period. Because the bare airframe was highly unstable, any amount of rate limiting was detrimental. The pilot for this example rated the nolimit baseline case HQRs of 5 and 4 (two evaluations) and PIORs of 3 and 2 because of pitch bobbles and abruptness.

The time history in Fig. 12 is typical of all of this pilot's runs with configuration 2DU with rate limiting. It is also a classic rate-limited PIO. A small PIO begins at about 3 s, with intermittent rate saturation of the actuator. Demand (commanded) is not much greater than achieved (actual) until around 10 s, where the PIO becomes divergent. At approximately 15 s in this segment, the safety pilot took over control of the airplane because the divergence became too large for flight safety.

Results for configuration 2DU are significant: This was a basically good-flying airplane, with no inherent characteristics indicative of a

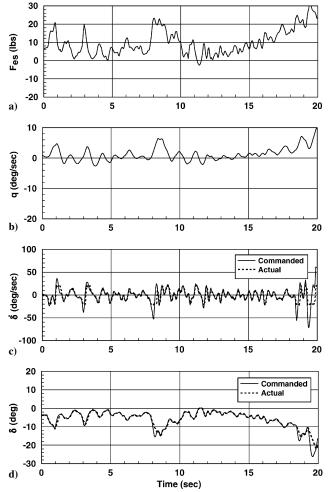


Fig. 10 Example time history segment for configuration 2D with 20-deg/s rate limit (no PIO tendencies).

PIO-prone design, that degenerated into a highly PIO-prone airplane solely in the presence of rate limiting. The cause, of course, is the use of active augmentation to provide the good-flying qualities on an airplane that otherwise was almost unflyable.

## Measures of Actuator Usage

The HAVE LIMITS data provide a base for assessment of actuator demands and for possible methods for avoiding excessive demands. Measures of actuator usage were made by computing cumulative actuator rates for the flight segments show in Figs. 10–12. Figure 13 shows a comparison of the cumulative actuator rates for two configurations (2D and 2DU) and two values of rate limiting. Actuator rate usage plots are not shown for configuration 2P. The plots for this case are very similar to those for 2D and do not show any significant new information.

The curves in Fig. 13 show actuator rates as a percentage of time for the 20-s run segments. The lowest curve in Fig. 13 is for configuration 2D with no added rate limit (labeled 2DU/157). (Note that 157 deg/s was the reported rate limit of the NT-33A's physical elevator surface.) For the 20-s time slice, actuator rates are below 20 deg/s for almost 90% of the run.

Just above the no-limit curve is the cumulative curve for the configuration 2D run of Fig. 10. Rate demands are almost unchanged from the no-limit case. The differences are probably due either to run-to-run variations in pilot gain, or to the slightly greater demand resulting from the small amount of time the actuator was on its rate limit. In either case, the pilot did not report any PIO tendencies.

The curves for configuration 2DU in Fig. 13 reflect the high demands on elevator for stabilizing the unstable airframe. For the unlimited case (2DU/157), actuator rate is below 20 deg/s for over 90% of the segment. For the final 10%, the demand increases sharply.

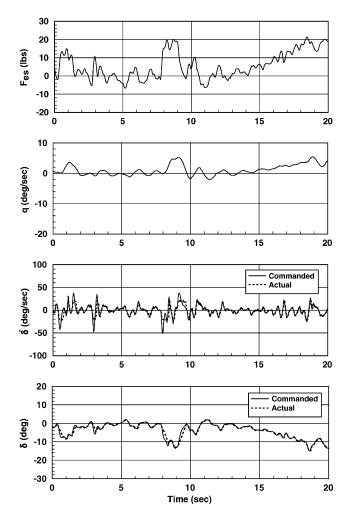


Fig. 11 Example time history segment for configuration 2P with 20-deg/s rate limit (tendency to PIO evidenced by phasing between stick force and pitch rate at t = 9-13 s).

When rate limiting at 20 deg/s is introduced, the demand cannot be met, the actuator is rate limited for most of the run, and a divergent PIO occurs.

#### **Pilot Rating Results**

The effects of rate limiting on pilot opinion for the HAVE LIMITS configurations are shown in Fig. 14. Mean Cooper–Harper HQRs and PIO tendency ratings are plotted against elevator rate limit. For the baseline configuration 2D, no pilot reported PIO, even when rate limits were as low as 10 deg/s, though handling qualities were degraded (mean HQR of 5). Because this configuration was slightly abrupt, a small amount of rate limiting actually improved the ratings, as evidenced by the dips in HQR and PIOR with a rate limit of 30 deg/s. Configuration 2P had degraded handling qualities and a slight tendency to PIO as rate limits were decreased. For configuration 2DU, any decrease in rate limits resulted in at least one divergent PIO.

#### **Pilot Rating Variability**

In the HAVE LIMITS flight experiment, the limited number of configurations (three, with seven rate limits) and pilots (three) did not result in significant variations in pilot opinion. A previous U.S. Air Force Test Pilot School project (HAVE GRIP<sup>15</sup>) found significant variations in pilot ratings and concluded that the evaluation task in that experiment (precision offset landing) was "insufficient to consistently uncover handling qualities deficiencies" resulting from rate limiting. What the authors of Ref. 15 actually discovered, however, was the variability in pilot opinion resulting from the introduction of a highly nonlinear phenomenon.

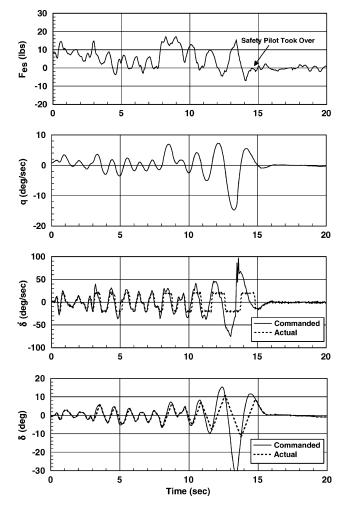


Fig. 12 Example time history segment for configuration 2DU with 20-deg/s rate limit (evidence of PIO starting at t=4 s, divergent at t=10 s).

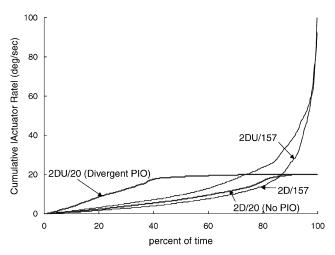


Fig. 13 Actuator rates for configurations 2D and 2DU (numbers on labels indicate rate limits in degrees per second).

A more recent ground-based simulation<sup>13,16</sup> confirmed that the nonlinear nature of rate limiting makes its impact very dependent on piloting technique. As an example, Fig. 15, reproduced from Ref. 16, shows the pilot ratings from nine different pilots for three configurations evaluated in the simulation.

The configuration labeled 2DR30 in Fig. 15 is the same as HAVE LIMITS configuration 2D, with a 30-deg/s rate limit (the case with the best pilot ratings from flight, Fig. 14). All nine pilots gave this

configuration good HQRs and PIORs. (Pilot B objected to a pitch bobble tendency.)

Configuration 1DL3 in Fig. 15 was a low-short-period model with added lags to make it susceptible to category 1 PIO; all pilots except pilot B reported at least one PIO with this configuration. Their HQRs and PIORs are reasonably consistent.

Configuration 2DUR20 is 2DU from HAVE LIMITS with a 20-deg/s rate limit. In the flight program (Fig. 14), this configuration was unflyable with divergent PIO. All three pilots in HAVE LIMITS assigned an HQR of 10 and a PIOR of 5 or 6. In the simulator, of the seven pilots who flew this configuration, six also experienced divergent PIOs and assigned HQRs of 10 and PIORs of 5 or 6. The seventh, pilot D, did not experience rate limiting or divergent PIO, and pilot D flew the configuration twice. In Refs. 13 and 16, an analysis of closed-loop control behavior shows that pilot D adopted the lowest bandwidth of all of the pilots.

If pilot D were the chief test pilot for an airplane manufacturer, it is conceivable that the simulator experiences with an airplane like configuration 2DUR20 could be convincing enough to persuade the company to build a prototype. Based on the experiences of the other six pilots, and of the three pilots in the HAVE LIMITS flight program, that would be a serious mistake.

## Alleviating the Effects of Rate Limiting

Despite the best intentions of flight control systems designers, actuator rate limiting may be inevitable for any aircraft. Even if the actuation system of a particular aircraft is designed to provide a margin from surface rate limiting, it is likely that at some point, changes to the software will result in the implementation of a software rate limiter, or changes in the aircraft's mission statement will result in a configuration that will overburden the surface rate limits. As this paper has demonstrated, for many aircraft the consequences of rate limiting will be negligible. In the past, when the consequences have not been negligible, and when it has been impractical to significantly change the control laws or the surface actuation system, solutions have been pursued through software. These solutions take the form of filters that are meant to reduce the likelihood of rate limiting, or to mitigate the negative effects if rate limiting is encountered. As such, the filters represent only a patch to minimize the existing deficiencies and not a real solution to the problem.

The following subsection presents a brief description of the more well-documented PIO alleviation methods. The different methods are described in general, without judgment as to their effectiveness. A comparative evaluation of these methods is beyond the scope of this study, but some discussion is still justified. The references cited provide additional information about each of the methods.

## **PIO Suppression Filters**

An observation from the Introduction of this paper is repeated: Nothing seems to inspire significant new handling qualities research like a high-profile PIO event. A PIO during landing on the space shuttle *Enterprise* approach and landing test (ALT) 5 in 1977 is a prime example. This PIO resulted from a combination of basic shuttle handling qualities, time delay through the digital flight control computers, and rate limiting of the elevator actuators.<sup>17</sup> Subsequent analysis and simulation<sup>18</sup> identified the unusual pilot location relative to the aircraft's center of rotation as a significant factor as well.

Methods for mitigating PIO on the space shuttle were investigated by NASA. In Ref. 17, a PIO suppression (PIOS) filter is described that was designed to reduce pilot gain when potential for PIO is high, while minimizing any additional phase lag. To achieve the desired gain reduction, the filter modifies the stick shaping function as a function of the amplitude and frequency of the pilot's input, thereby reducing the amount of rate limiting.

The PIOS filter was implemented in the shuttle control laws, and no pitch PIOs have been reported in the open literature since the 1977 ALT-5 event. Based on the apparent success of the application of the shuttle PIOS filter, two types of filters were evaluated by NASA Dryden Flight Research Center in three flight-test programs. <sup>19</sup> These evaluations found that PIOS filters can improve handling qualities of fighter aircraft that are susceptible to PIO because of excessive

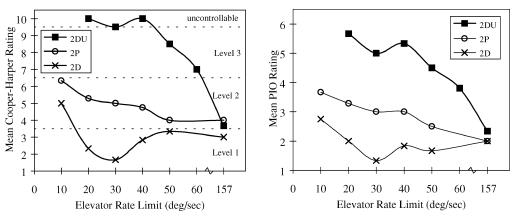


Fig. 14 Mean Cooper-Harper HQRs and PIO tendency ratings for HAVE LIMITS configurations.

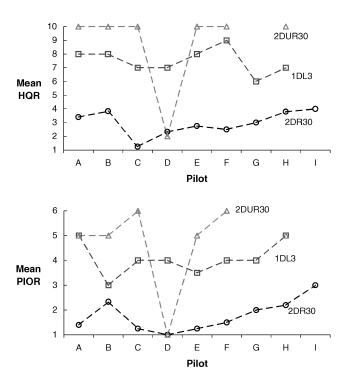


Fig. 15 Pilot rating variability (reproduced from Ref. 16).

time delay. Of course, such delays are the primary degrading effect of rate limiting.

## U.S. Navy's Phase Compensating Rate Limiter

In the late 1980s, engineers with the Naval Air Systems Command of the U.S. Navy developed a phase compensating filter to reduce the effects of rate limiting.<sup>20</sup> The filter is time-domain based and reverses the direction of the actuator when 1) the actuator is on its rate limit and 2) the actuator is traveling in opposite direction from its command. This filter, patented by the U.S. government, is similar to that tested by NASA on variable-stability aircraft in the 1990s.<sup>21</sup> Some of the problems encountered with that version of the filter include sensitivity to input noise and a tendency to develop an offset bias over time. Most of the problems with the filter can be overcome with additional complexity. A modified version on the U.S. Navy's compensation method has been implemented on the F-18E/F aircraft.

#### SAAB System

The phase compensation technique developed by SAAB to overcome the deficiencies associated with rate limiting on the JAS-39 Gripen<sup>5</sup> uses a feedback signal through a low-pass filter that will almost immediately reverse the direction of the rate limit output when the input reverses direction. The technique also features a by-

pass circuit that ensures that only the low-frequency components of the input are limited with phase compensation. Unlike the U.S. Navy's phase compensating rate limiter, the Gripen design introduces some additional effective delay through the lag filter. After successful simulation and flight-test evaluations, the rate limiters with feedback and bypass were qualified for use in the Gripen production flight control system software.

#### **DLR German Aerospace Research Center System**

One final rate-limiting alleviation scheme to discuss is a phase compensation technique developed by DLR, German Aerospace Research Center. As described in Ref. 22, a simple phase compensation algorithm for a rate-limiting element is derived directly from the describing function relationships of Hanke<sup>8</sup> shown earlier in this paper. The compensator does not use feedback or logic, but instead reduces the input amplitude as a function of frequency by the same amount that the amplitude is reduced by the rate limiter. Hence, the rate limiter can follow the input signal without added phase lag. The concept has been successfully demonstrated in-flight for a wide range of system inputs using the DLR ATTAS (advanced technologies testing aircraft system).

## **Conclusions**

- 1) Rate limiting of control effector actuators can have dire effects on handling qualities and PIO. The magnitude of the effects depends on three basic factors: a) how long the actuator is rate limited, b) how much more the pilot/flight control system demands of the airplane, and c) the consequences on aircraft dynamics of encountering the limit.
- 2) Rate limiting can cause PIO. Fundamentally, rate limiting introduces phase lag into the aircraft's response. This alone can be sufficient to lead to PIO, depending on the characteristics of the aircraft. Flight 1–1–5 of the X-15 provides a perfect example of this effect. If the aircraft is augmented, reaching a rate limit also means a sudden change in response dynamics as the aircraft transitions from the closed-loop to the open-loop response. If the open-loop response is unstable, divergent PIO is likely, especially when the additional phase lag is introduced.
- 3) Rate limiting does not necessarily cause PIO. If the response characteristics of the basic aircraft are of sufficiently high bandwidth, the additional phase loss due to rate limiting will not lead to PIO.
- 4) PIO can lead to rate limiting. Although not specifically addressed in this paper, it is apparent that if an aircraft is susceptible to PIO even when there is no rate limiting the occurrence of PIO can push control surface effectors onto their rate limits.
- 5) Rate limiting is a nonlinear phenomenon and, hence, is highly dependent on pilot technique. It is possible for two pilots to have vastly different opinions of the same aircraft simply depending on their control strategies. Despite our best attempts at devising consistent, repeatable evaluation tasks, some pilots will naturally adopt

strategies that will minimize the chance of ever encountering the rate limiting. This is why it is critically important that aircraft, and highly augmented aircraft especially, be evaluated by as many pilots as possible.

6) Software-based alleviation methods may help reduce the adverse effects of rate limiting, but they should be considered patches and not solutions. Recent efforts to develop methods for reducing the phase loss due to rate limiting appear to show promise. Whereas full evaluation of these methods is beyond the scope of this study, published reports suggest that the more well-documented methods have proven effective at reducing the effects of rate limiting. Still, such methods should always be viewed as patches, not design solutions.

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## **Economic Principles Applied to Space Industry Decisions** Joel S. Greenberg, Princeton Synergetics, Inc.



This is not an economics book. It is a book about the application of economic principles and concepts in decision making related to space activities. The book is primarily tutorial and elaborates upon concepts and methodology and their applications. Emphasis is placed upon applications with typical results of performed analyses presented to demonstrate concepts and methods.

The use of mathematical and simulation models serves as the underpinning for much of the presented materials. The specific models considered have been selected to demonstrate the role that a structured thought process can play in the decision process. Since most decisions relating to technology development, product design, capital expenditures, and investments involve uncertainty and risk, a number of the selected models, developed methodologies, and presented examples explicitly and quantitatively consider uncertainty and risk.

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